

**研究論文**

# GMA 용접에 최적의 용접비드 형상을 예측하기 위한 수학적 모델 개발

김 일 수\*

## A DEVELOPMENT OF MATHEMATICAL MODELS FOR PREDICTION OF OPTIMAL WELD BEAD GEOMETRY FOR GMA WELDING

I. S. Kim\*

**Key Word** : GMA Welding(가스 금속아크용접), Process Variables(공정변수), Welding Bead Geometry(용접비드형상), Mathematical Modeling(수학적 모델), Factorial Experiment Method(요인사험 방법)

### ABSTRACT

With the trend towards welding automation and robotization, mathematical models for studying the influence of various variables on the weld bead geometry in gas metal arc (GMA) welding process are required. Partial-penetration, single-pass bead-on-plate welds using the GMA welding process were fabricated in 12mm mild steel plates employed four different process variables. Experimental results has been designed to investigate the analytical and empirical formulae, and develop mathematical equations for understanding the relationship between process variables and weld bead geometry. The relationships can be usefully employed not only for open loop process control, but also for adaptive control provided that dynamic sensing of process output is performed.

### 1. INTRODUCTION

GMA welding involves large number of interdependent variables that can affect product quality, productivity and cost effectiveness. The

relationships between GMA welding variables and weld bead geometry is complex because of the number of variables and their interrelationships involved. Variables such as capacity and type of equipment and material composition are considered to be fixed, while primary adjustable

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\* 정희원, 목포대학교 기계공학과

variables such as arc voltage, welding current and welding speed can be altered during GMA welding process. Many attempts have been made to predict and understand the effect of the welding variables on the weld bead geometry. These include the entirely theoretical studies based on heat flow theory<sup>1-6)</sup> and the empirical methods based on studies of actual welding applications<sup>7-15)</sup>. An early approach to procedure optimization called, "tolerance box approach", was developed to optimize welding procedure selection<sup>8-9)</sup>. Welding variables were optimized using the tolerance to process variations and production rate, but this approach is difficult to implement in process control situations when dealing with more than three input variables.

The above mentioned work has been summarized by Shinoda and Doherty<sup>10)</sup>. McGlone<sup>11)</sup>, and McGlone and Chadwick<sup>12)</sup> have reported a mathematical analysis correlating welding variables and weld bead geometry for the submerged-arc welding of square edge close butts. Process variables in these studies included welding current, arc voltage, welding speed, bevel angle and electrode diameter. Similar mathematical relationships between welding variables and fillet weld geometry for GMA welding using flux-cored wires have also been reported<sup>13)</sup>. Chandel<sup>14)</sup> first applied this technique to the GMA welding process and investigated the relationship between process variables and weld bead geometry. These results showed that arc current has the greatest influence on weld bead geometry, and that mathematical models derived from experimental results can be used to predict weld bead geometry accurately. Yang, et al.<sup>15)</sup> further extended this study to the weld deposit area and presented the effects of electrode polarity, extension and diameter, welding current, arc voltage, travel speed, power source setting and flux basicity on the weld deposited area.

Most conventional real-time controlled welding systems attempt to control major output

parameters such as weld bead width. Hunter et al.<sup>16)</sup> not only employed logarithmic equations to model the GMA welding variables, but also adapted capacitive distance transducers for gun positioning and ultrasonic sensing to monitor wire stickout. Richardson et al.<sup>17)</sup> have been employed an optical sensor coaxially mounted with a GTA welding electrode to provide a pattern of reflected arc light for a computer algorithm to interpret joint width and location, and give successful joint tracking. Doumanidis et al.<sup>18)</sup> have attempted to derive simple dynamic models in their attempt to control weld bead width, penetration, heat affected zone and rate of cooling at the centerline of the weld. Even if more recently statically designed experiments and compatible analytical techniques have been implemented to establish relationships between process variables and welding bead geometry, both techniques have proven to possess only a limited range of applicability, but not lead a multivariable control system application<sup>19)</sup>.

The objectives of this study were to model weld bead geometry, to compare the experimental results to outputs obtained using the sets of the existing formulae in the literature relating input variables to output variables and to finally develop a mathematical model explaining the relationship between GMA process variables and weld bead geometry of bead-on-plate welds.

## 2. EXPERIMENTAL PROCESS

The 3n experimental factorial design provided the smallest number of treatment combinations where n factors were studied in a complete factorial arrangement, and not only the main effect of each factor, but also the interactions between these factors<sup>20,21)</sup>. In this study, results were used only for fitting the response curve. The chosen factors are wire diameter (D), arc voltage (V), welding current (I) and welding speed (Vw). The response is weld bead width (W), height (H) and

penetration (P). The  $2 \times 3^3$  factorial design provided the main effect and interaction effects of four variables at two or three levels. The factorial design required 54 weld runs for fitting each equation. The welding variables and limits employed are given in Table 1.

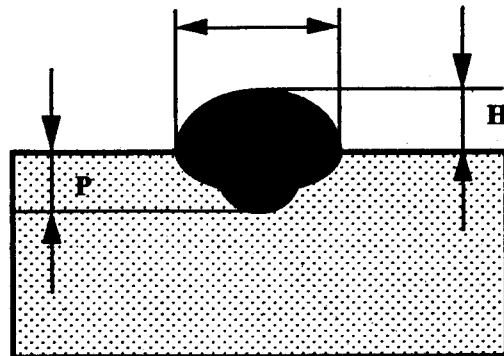
**Table 1.** Welding variables and limits employed

Variables	Units	Limits		
		Low	Middle	High
D	mm	1.2		1.6
V	Volt	20	25	30
Vw	cm/min	25	33	41
I	Ampere	180	260	360

The welding facility at the Intelligent Control Lab. in Mokpo National University was chosen as the basis for the data collection and evaluation. The facility consists of a GMA welding unit which includes a welding power source, welder remote control unit and wire torch, and a six-axis robot manipulator that has a robot control unit and robot teach box. Torch positioning and motion control were obtained using the robot controller. The selection of the electrode wire was principally based upon matching the mechanical properties and physical characteristics of the base metal, weld size, and existing electrode inventory. Steel wires of 1.2 and 1.6 mm diameter with composition C 0.07-0.15 %, Mn 1.00-1.50 %, Si 0.60-0.85 %, S 0.035 % max, P 0.025 % max and Cu 0.5 % max, were used. To achieve equilibrium during GMA welding process, two samples were taken for observation after discarding 50 mm on each side to eliminate the end effects. Welding was carried out on experimental plates of  $200 \times 75 \times 6$  mm mild steel flats adopting the bead-on-plate technique.

Experimental test plates were located in the fixture jig by the robot controller and the required input conditions were fed for the particular weld

steps in the robot path. With welder and argon shield gas turned on, the robot was initialized and welding was executed. This procedure was followed until the factorial experimental design runs were completed. After 54 welds, the plates were cut using a power hacksaw and the end faces were machined. Specimen end faces were polished and etched using a 2.5% nital solution to reveal grain boundaries and display the depth of penetration. A profile projector with the image magnification of 10 and 20 times was used to accurately measure the weld bead geometry. The results of the experiment were analyzed on the relationships between input variables and output variables of GMA welding process. Fig. 1 defines the weld bead geometry studied.



W : weld bead width  
H : weld bead height  
P : weld bead penetration

**Fig. 1** Weld bead geometry

### 3. ANALYSIS OF RESULTS

#### 3.1 Comparison with theoretical Models

Theoretical predictions of weld bead geometry can be predicted from conductive heat transfer studies due to simplicity and minimization of the

calculated time for real-time control. These assume that the weld completely penetrates the plate being welded and heat is conducted only in the plane of the plate. Roberts and Wells<sup>2)</sup> have estimated the weld bead width to be given by:

$$W = \frac{1}{2} \frac{Q}{Vwt} \frac{1}{\rho C} \frac{1}{T} - \frac{4}{5} \frac{\lambda}{Vw} \quad (1)$$

where

$Q$  = rate of heat input

$t$  = plate thickness

$T$  = melting temperature - ambient temperature

$W$  = weld bead width

$\rho C$  = volumetric heat capacity

$\lambda$  = thermal diffusivity

The weld bead penetration was assumed to be equal to a weld bead half-width, a semicircular cross section. Values of the material parameters were assumed to be  $\lambda = 0.091\text{cm}^2/\text{sec}$ ,  $\rho C = 4.5\text{J}/\text{cm}^3\text{C}$ ,  $T = 1500\text{C}$ ,  $k = 0.41\text{J}/\text{cmCsec}$ . In GMA welding, the rate of heat input to plate, is given by the product of  $\zeta$  and  $I$ . Heat input efficiency,  $\zeta$  for the GMA welding process employed to weld steel plates is based on process variables such as arc voltage, welding current, electrode extension and type of shielding gas. It was assumed to be 68% for the comparisons made below. The experimental weld bead width was generally larger than the theoretical one, while the experimental weld bead penetration was usually smaller than the theoretical one. Linear regression analysis was used to compare the experimental with theoretical results for the two-dimensional (2D) mathematical model. The results obtained are as follows:

$$W_1 = 8.63374 + 0.5494W \quad (2)$$

$$P_1 = 0.8394 + 0.5527P \quad (3)$$

where

$W_1$  = predicted width

$P_1$  = predicted penetration

Table 2 shows the standard error of estimate,

coefficient of multiple correlation, and coefficient of determination for the above described models accordingly. The values of coefficient of multiple correlation of these equations are 0.702 and 0.6035 respectively. It is noted that the coefficient of multiple correlation for equation (2) is higher than that for equation (3). The scatter graphs of measured and calculated values of weld bead geometry are illustrated in Figs. 2 and 3. The line of best fit for the plotted points was drawn using linear regression computation. From Figs. 2 and 3, it is found that equations (2) and (3) have an inclination to overestimate the weld bead penetration, whereas the weld bead width was overall slightly larger than the values obtained during experimentation. Christensen et al.<sup>3)</sup> have presented similar findings. Also, Friedman and Glickstein<sup>4)</sup> have analytically shown that a larger diameter heat source tends to increase weld bead width and decrease weld bead penetration in stationary GTA welding.

**Table 2.** Analysis of models using Roberts and Wells equation

No. of model	Standard error estimate	Coefficient of multiple correlation	Coefficient of determination
Weld bead width	0.50234	0.7020	0.6962
Weld bead penetration	0.67543	0.9056	0.8979

Christensen et al.<sup>3)</sup> have published theoretical non-dimensional graphs showing various weld bead dimensions versus "operating parameters",  $n$ , where  $n = \frac{QVw}{4\pi\lambda kT}$ . Their model is claimed for being suitable to all combinations of materials and welding conditions within the limitations and assumptions combined with the point source equation. The experimental results were plotted using the same non-dimensional parameters, and compared with the theoretical results obtained by Christensen et al.<sup>3)</sup>, which assumed a three-

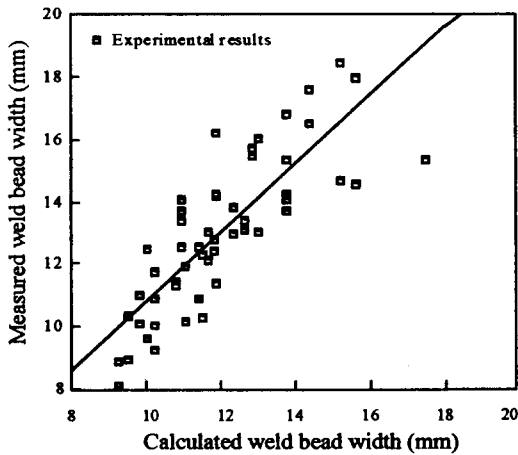


Fig 2. Comparison between measured and calculated weld bead width

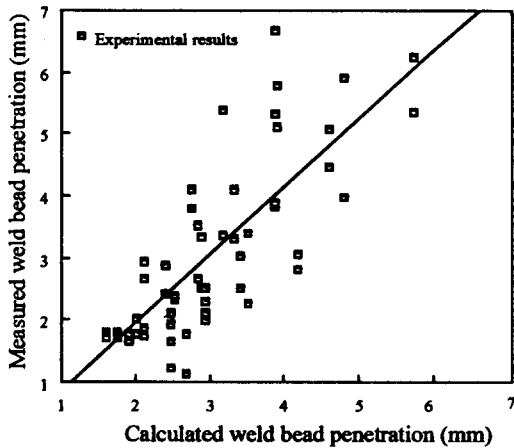


Fig 3. Comparison between measured and calculated weld bead penetration.

dimensional (3D) conductive heat transfer configuration. Fig. 4 shows the non-dimensional weld bead width. Reasonable agreement between the experimental and the theoretical non-dimensional weld bead width is shown, even when the scatter about the theoretical results is considerable. From Fig. 5, it appears that the theoretical equation also overestimates the weld bead penetration.

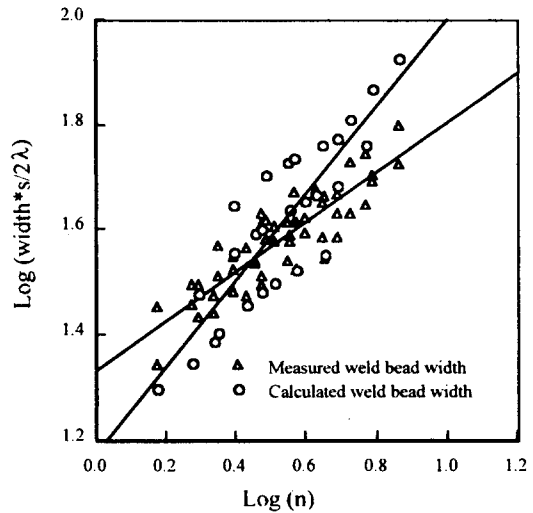


Fig 4. Non-dimensional weld bead measurements versus operating parameter( $n$ ) for weld bead width.

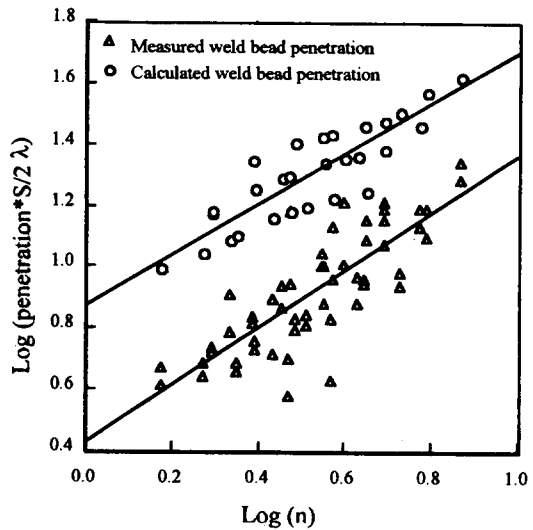


Fig 5. Non-dimensional weld bead measurements versus operating parameter( $n$ ) for weld bead penetration.

In addition, it is quite evident from the above comparison that prediction of weld bead geometry with reasonable accuracy, based on various models, requires adjustments in order to achieve

better agreement with experimental results. Since conductive, convective, radiative heat transfer and mass transfer in the GMA welding process all take their toll, the development of an accurate analytical model can be complicated and perhaps inappropriate for either closed loop or adaptive control purposes. Instead, a regression model for weld bead geometry should be considered.

### 3.2 Comparison with Empirical Models

The empirical equations reported by Chandel<sup>14</sup> for GMA welding processes were also employed to predict weld bead geometry. The design matrix and the treatment combinations used to produce the 54 weld runs for fitting each equation were input into the above empirical equations to provide theoretical results for weld bead width, height and penetration accordingly. This allowed the accuracy of Chandel's equations to be compared to experimental results. Results were plotted using a scatter graph for each weld bead width, height and penetration. Three graphs were produced for experimental versus theoretical results using Chandel's equations. These three graphs are presented in Figs. 6 to 8 where the theoretical results are depicted on the X axis and experimental results on the Y axis, with a line of best fit drawn for the plotted points, using regression computations. It was quite evident from results that none predicted the experimental values with reasonable accuracy. However, when these values were plotted as data points in a scatter graph, a definite correlation appeared.

## 4. DEVELOPMENT OF MATHEMATICAL MODELS

In order to quantitatively evaluate the effect of process variables on the weld bead geometry, the mathematical relationship between process

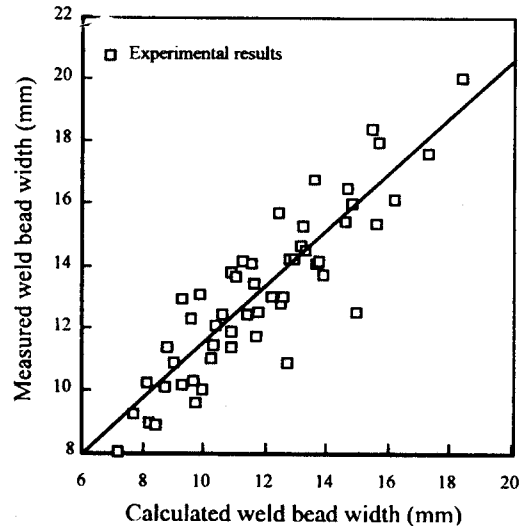


Fig 6. Comparison between measured and calculated weld bead width using Chandel's equations.

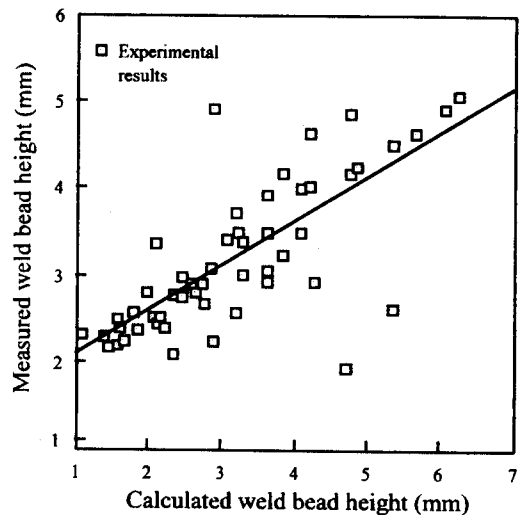


Fig 7. Comparison between measured and calculated weld bead height using Chandel's equations.

variables and weld bead geometry have been developed. In general, the response function can be represented as follows:

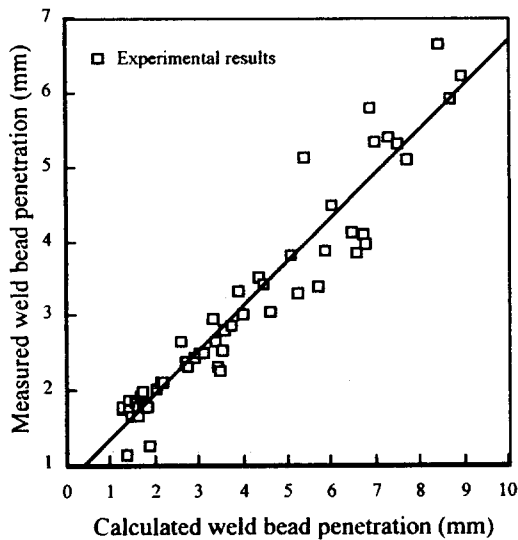


Fig 8. Comparison between measured and calculated weld bead penetration using Chandel's equations.

$$Y = f(D, V, I, V_w) \quad (4)$$

According to McGlone and Chadwick<sup>12)</sup> the above equation can be expressed as follows;

$$Y = a(D)^b(V)^c(I)^d(V_w)^e \quad (5)$$

where empirical coefficients  $a, b, c, d$  and  $e$  are constants and depend on the gas flow rate, wire stickout and material type. The values of  $a, b, c, d$  and  $e$  were computed by the method of regression<sup>20, 21)</sup>. These analyses were carried out with help of a standard statistical package, SAS<sup>22)</sup>. The following equations correlating process variables and weld bead geometry were obtained from the experimental results;

$$W = (D^{0.4294}V^{-0.7083}I^{0.3518}V_w^{-0.4590})10^{-0.0905} \quad (6)$$

$$H = (D^{0.1255}V^{-0.7183}I^{0.6387}V_w^{-0.2395})10^{0.3339} \quad (7)$$

$$P = (D^{-0.5668}V^{0.0130}I^{1.4005}V_w^{-0.3641})10^{-2.3098} \quad (8)$$

The adequacy of the models and the significance of coefficients were tested by applying the analysis of variance technique respectively. Table 3 shows the standard error of estimates, coefficients of multiple correlation, and coefficients of determination for the above models respectively. It is evident that all models were adequate. The mathematical models were employed to calculate the theoretical results of regression analysis and to compare the experimental results measured with the set of published formula reported by Chandel. To ensure the accuracy of the new equations and survey the spread of the values, results were again plotted using the scatter graph. These graphs of experimental versus theoretical values of weld bead dimensions are presented in Figs. 9 to 11 for weld bead width, height and penetration respectively. During analysis of the results, it was observed that the mathematical models for GMA welding yielded relatively accurate weld bead dimensions.

To make effective use of automated and robotic arc welding, it is imperative that mathematical models are developed which can be programmed easily and fed to the robot controller having a high degree of confidence in predicting the weld bead geometry and shape relations to accomplish the desired mechanical properties of the weldment. They should also cover a wide range of material thicknesses and be applied for all position welding. For the automatic, open loop control, welding system to use this data, the data must be available in the form of mathematical equations. It was in the light of these concluding remarks and

Table 3. Analysis of variance test for mathematical model

No. of model	Standard error estimate	Coefficient of multiple correlation	Coefficient of determination
Weld bead width	0.85437	0.9398	0.9259
Weld bead height	0.73431	0.9037	0.8960
Weld bead penetration	0.67543	0.9056	0.8979

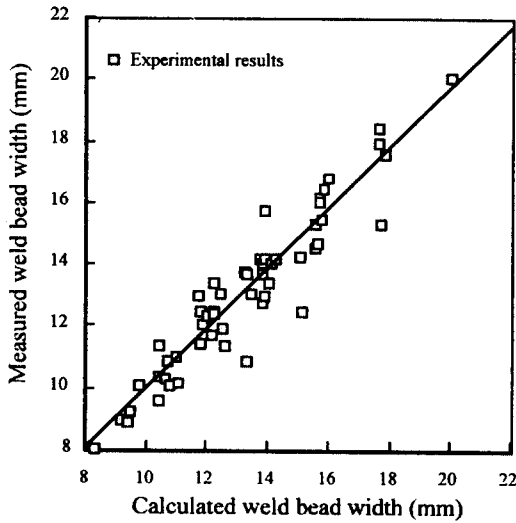


Fig 9. Comparison between measured and calculated weld bead width using mathematical equations.

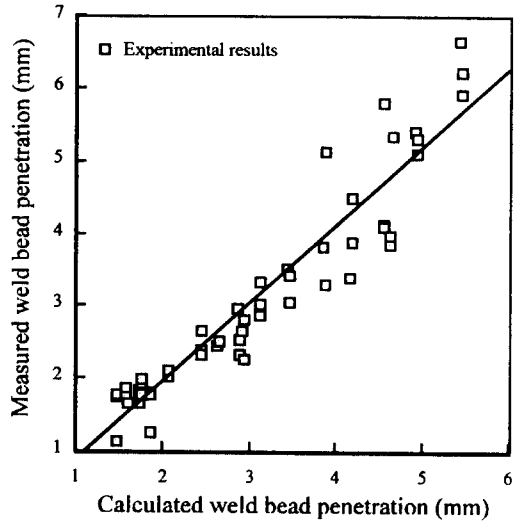


Fig 11. Comparison between measured and calculated weld bead penetration using mathematical equations.

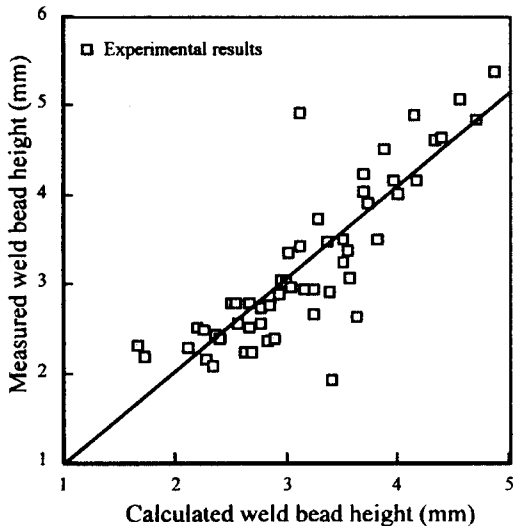


Fig 10. Comparison between measured and calculated weld bead height using mathematical equations.

suggestions for further developments outlined by previous researchers that the work in this paper was undertaken.

## 5. CONCLUSIONS

The effects of process variables on weld bead geometry when bead-on-plate welds are deposited using the GMA welding process have been studied and the following conclusions reached:

1. The universality of results obtained after using empirical equations taken from existing models developed by Chandel proved to be limited in predicting experimental bead shapes for the GMA welding process.
2. Comparison between weld bead geometry experimental findings and those proposed by the conductive heat transfer model showed that the theoretical analysis generally overestimates weld bead penetration and suitably predicts the weld bead width, even if considerable scatter is found in the overall results.
3. Mathematical models developed from the observed data in the course of this work can be used to control the process variables in order to achieve desired weld bead geometry outcomes and



indeed weld quality.

Although the empirical formulae based on experimental results are valid for current process variables and bead geometry, it is proposed that these equations are extended to plates of varying thickness and many other variables which is not included in this research.

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